Master MMMEF, 2021-2022
Final Exam on:
General Equilibrium Theory:
Economic analysis of financial markets
December 2021
2 hours

December 16, 2021

Q1) In a two-period model with uncertainty, explain the non satiation state by state assumption for the utility function of a consumer.
Q2) What is the relationship between an equilibrium with a full set of Arrow securities and a contingent commodity equilibrium?
Q3) In a two-period model, given a financial structure represented by the payoff matrix function $p \to V(p)$, what is the definition of the full payoff matrix ?
Q4) Give a necessary condition on the financial structure to get the same equilibrium allocations for the financial equilibrium and for the contingent commodity equilibrium.
Q5) With a nominal financial structure represented by the matrix V and an arbitrage free asset price q, what is a present value vector associated to q?

Exercise 1 We consider a two-period model with the uncertainty represented by the graph $\mathbb{D}$ and a financial structure with J assets represented by the constant $\sharp\mathbb{D}_1 \times J$ payoff matrix V. We assume that the financial structure has no useless portfolio. Show that the vector $q \in \mathbb{R}^J$ is arbitrage free for the financial structure if and only if $q \cdot z > 0$ for all $z \in \mathbb{R}^J \setminus \{0\}$ such that $V(z) \geq 0$.

Exercise 2 We consider a two-period model with the uncertainty represented by the graph $\mathbb{D} = \{\xi_0, \xi_1, \xi_2\}$ where ξ_1 and ξ_2 are the two successors of ξ_0. There is a unique commodity at each state and the price of the commodity on the spot market is normalized to 1. There are two consumers with the same utility function:

$$u(x_0, x_1, x_2) = x_0 x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

The initial endowments are: $e^1 = (3, 1, 1)$ and $e^2 = (\frac{1}{3}, 3, 2)$.

We first assume that there is a unique asset (the riskless bond) on the financial market with the payoffs $(1, 1)$. We denote by $q > 0$ the price of this asset.

1) Write explicitely the utility maximisation problem over the financial budget set for both consumers.

2) Show that the above problem for the first consumer can be reduced to the following one where z^1 is the unique unknown:

$$\max\{(4 - qz^1)(1 + z^1)^{\frac{1}{2}}(1 + z^1)^{\frac{1}{2}} \mid z^1 \in [-1, \frac{4}{q}]\}$$

and write the equivalent problem for the second consumer with the quantity of asset as unique unkown.

3) Show that for $q = 1$, $z^1 = 1$ and $z^2 = -1$ are solutions of the two above problem. Deduce a financial equilibrium of this economy. Is the equilibrium allocation Pareto optimal?

We now assume that there is a second asset (an Arrow security) on the financial market with the payoffs $(1, 0)$.

4) Show that the financial structure is complete with these two assets.

5) Give the definition of a contingent commodity equilibrium in this economy.

6) Show that $(\pi = (1, \pi_1, \pi_2), x^{*1} = (x_0^{*1}, x_1^{*1}, x_2^{*1}), x^{*2} = (x_0^{*2}, x_1^{*2}, x_2^{*2}))$ is a contingent commodity equilibrium if:

$$\begin{cases} \frac{x_0^{*1}}{2x_1^{*1}} = \frac{x_0^{*2}}{2x_1^{*2}} = \pi_1 \\ \frac{x_0^{*1}}{2x_2^{*1}} = \frac{x_0^{*2}}{2x_2^{*2}} = \pi_2 \\ x_0^{*1} + \pi_1 x_1^{*1} + \pi_2 x_2^{*1} = 3 + \pi_1 + \pi_2 \\ x_0^{*2} + \pi_1 x_1^{*2} + \pi_2 x_2^{*2} = \frac{1}{3} + 3\pi_1 + 2\pi_2 \\ x_1^{*1} + x_1^{*2} = 4 \\ x_2^{*1} + x_2^{*2} = 3 \end{cases}$$

7) Check that $\left((1, \frac{5}{12}, \frac{5}{9}), \frac{143}{240}(\frac{10}{3}, 4, 3), \frac{97}{240}(\frac{10}{3}, 4, 3)\right)$ is the contingent commodity equilibrium. Is this allocation Pareto optimal?

8) Give a financial equilibrium of the economy with the two assets.