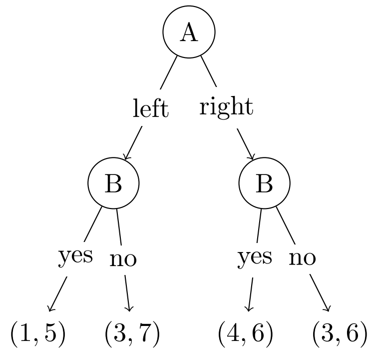
Multi-Agent Systems · Exam

Université Paris I (Panthéon-Sorbonne) · February 25th, 2022

- Write your solutions in **English**.
- You cannot use any connected device for the entire duration of the exam.
- You can only consult two A4 pages of notes.
- Always justify your answers by showing and explaining your calculations and reasoning.

Question 1

Consider the following tree representation of a two-player perfect-information extensive-form game. Player A moves first and can choose between actions 'left' or 'right', and player B can respond by playing actions 'yes' or 'no'. The utilities for the two players are indicated on the leaf nodes.



a. Provide the payoff matrix of the corresponding normal-form game.

b. Find all the pure-strategy Nash equilibria and all the Pareto-optimal profiles of the game.

c. Find all the subgame-perfect equilibria of the game.

Question 2

For each of the following facts, say whether they are true or false. If a fact is true, give a formal argument to explain why; if a fact is false, give a counter-example to show why not.

a. If a normal-form game is zero-sum, then all profiles are Pareto optimal.

b. If in a normal-form game all profiles are Pareto optimal, then the game is constant-sum.

Question 3

Consider a voting setting S for two voters $\mathcal{N} = \{1, 2\}$ and two outcomes $O = \{a, b\}$. Assume that agents submit a profile of strict preferences $[\succ]$ over O and consider *resolute* social welfare functions.

a. How many different social welfare functions in S satisfy *Pareto efficiency*?

b. How many different social welfare functions in S are *independent of irrelevant alternatives*?

c. How many different social welfare functions in S are *non-dictatorial*?

d. Can you find a social welfare function in S which satisfies Pareto efficiency, independence of irrelevant alternatives, and non-dictatoriality?

Question 4

Consider an instance of two-sided matching, where the preferences for the ten agents into the two sets $X = \{a, b, c, d, e\}$ and $Y = \{1, 2, 3, 4, 5\}$ are as follows:

a	$4 \succ 2 \succ 3 \succ 5 \succ 1$		1	$a \succ b \succ d \succ c \succ e$
b	$4 \succ 2 \succ 1 \succ 3 \succ 5$		2	$a \succ b \succ c \succ d \succ e$
c	$2 \succ 1 \succ 5 \succ 4 \succ 3$		3	$c \succ a \succ b \succ e \succ d$
d	$2 \succ 4 \succ 5 \succ 1 \succ 3$		4	$c \succ d \succ e \succ a \succ b$
e	$2 \succ 4 \succ 1 \succ 5 \succ 3$		5	$d \succ b \succ a \succ c \succ e$

a. Does there exist a stable solution for the instance above? If yes, is it unique?

Consider now an instance of one-sided matching, where the agents in X have preferences as above over the objects in Y, and the initial assignment is $(a, 1), (b, 5), (c, 4), (d, 2), (e, 3)$.

b. Compute the outcome of the Top-Trading Cycles mechanism on this instance.

Question 5

Consider the setting of single-good auctions for one seller and multiple buyers.

a. Recall the allocation rule and the payment rule of a Vickrey auction.

b. Briefly explain why in a Vickrey auction it is always a dominant strategy for a buyer to bid their true private value for the object.

Question 6

Consider the following participatory budgeting instance, for five agents $\mathcal{N} = \{a_1, a_2, a_3, a_4, a_5\}$ and four projects $P = \{p_a, p_b, p_c, p_d\}$ whose costs are $c(p_a) = c(p_b) = 2$, $c(p_c) = 3$ and $c(p_d) = 4$. The budget limit is fixed at $b = 5$ and agents express their preferences via the approval ballots shown below:

	p_a 2	p_b 2	p_c 3	p_d 4
a_1			✓	✓
a_2		✓		✓
a_3	✓	✓		✓
a_4	✓	✓		✓
a_5	✓		✓	

a. Choose two rules among the ones we saw in class and compute their outcome on this profile.

b. Do the two outcomes differ? Do they maximize the agents' satisfaction (for the chosen definition)?

(Questions 2 and 3 were originally proposed by Ulle Endriss and Fernando R. Velázquez-Quesada).